The effect of chamfer and size on the stress distributions in a thick-walled cylinder with a cross bore under internal pressure

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(Recieved February 1995; Final version July 1995)

Abstract

The hoop stress and stress concentration factor distributions in a closed ended thick-walled cylinder with a chamfered cross bore under internal pressure were investigated. The finite element method of stress analysis was used for this study. The effect of changing chamfer angle, for a fixed chamfer size, on the hoop stress and stress concentration factor distributions was investigated. The effect of changing the chamfer length, for a fixed chamfer angle, on the hoop stress distributions and stress concentration factors was also investigated. The optimum chamfer angle and chamfer length for any cylinder configuration was established. The study revealed that adding chamfers to cross bores causes a redistribution and reduction of the stresses attained in internally pressurized thick-walled cylinders. For a thick cylinder of a given thickness ratio and cross bore diameter, it was observed that the stresses were a minimum at a specific chamfer angle and size.

Nomenclature

ASME	American Society of Mechanical Engineers
K_s	ratio of main bore radius to cross bore radius

Introduction

In the nineteenth century, bursting of boilers was a powerful incentive for research in pressure vessel design.[10] At the beginning of this century, boiler explosions in the United States averaged one per day.[11] Developments in large chemical vessels and rocket engines have made more urgent the need for accurate and reliable solutions for pressure vessel designs in order to avoid the catastrophes like those cited above.

Pressure vessels can at times be subjected to extremes of operating conditions. For instance, forged high pressure reactors with thickness ratios of two and above have been used extensively for stirred autoclaves in the manufacture of low density polyethylene production where the operating pressures are as high as 250 MPa, and the temperatures close to 300°.[9] Forged high pressure vessels have also been used for isostatic compaction of metallic and ceramic powders at pressures of 200 to 300 MPa.[5] Pressure vessels hold immense potential energy exerted by the working fluid and therefore human and material resource safety is very important in the design of vessels.

It is inevitable that pressure vessels have openings on their ends or sides for the following purposes: fluid temperature measurement fittings, internal pressure measurement, bursting discs, inspection covers, relief and safety valves, gas inlets, etc.[13] These features introduce geometric discontinuities to the cylinder configurations. When the vessel is pressurized, the intersection of the cross bore and the cylinder surfaces form stress singularity points. These stress concentrations reduce the pressure carrying capacity of such vessels below that of a plain cylinder without cross bores.[13;7] A proper understanding of the stress severity in these regions of high stress fields would lead to usage of low safety factors in the design of such vessels, improved plant availability and enhanced safety.

In practice, this problem of high localized stresses has been overcome by forming a radius at this intersection. Ford *et al.* [5] suggested the introduction of a carefully polished chamfer at this intersection, which can be obtained using spark erosion techniques. This was based on experience rather than analysis.

For high pressure applications, a realistic picture of the state of stress in a vessel with side ports is needed because fatigue life is very critical and present day limitations of strength and ductility in commercial pressure vessel materials prevent high factors of safety.[2] It is clear that section I and Division I of section VIII of the ASME Boiler and Pressure Vessel code do not call for a detailed stress analysis but merely set the wall thickness necessary to keep the basic hoop stress below the tabulated allowable stress. The higher localized stresses are taken care of by the safety factor and a set of design rules [1] and hence there is a need for determining the state of stress in this region by proper analysis.

Analytical procedure

The finite element method of three-dimensional stress analysis was applied in this study. The closed ended thick-walled cylinders were represented by only a quarter of the structure due to symmetry. Hence only a quarter of the cylinder was considered. This has far reaching advantages in the finite element method in that computer storage requirements are reduced by 75% and the computer (CPU) run time is also highly minimized.

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In order to observe the variation of hoop stress profiles and the stress concentration factors in the body of the cylinder material, the cases listed below were considered for an elastostatic Hookean material:

(a) Cross bores of increasing diameter were analysed for hoop stresses and stress concentration factor.

(b) Cross bores of increasing chamfer length and fixed chamfer angle were analysed for hoop stresses and stress concentration factor.

(c) Cross bores of increasing chamfer angle and fixed chamfer length were analysed for hoop stresses and stress concentration factor.

(d) For a given cross bore radius, a plain cross-bored cylinder was considered in order to observe and compare the level of hoop stresses and stress concentration factor.

(e) A plain thick walled cylinder without a cross bore was analysed for stresses in order to verify the results of the above cases.

A standard finite element analysis approach was used to generate the global co-ordinates for all the nodes and the nodal connectivity matrix for all the elements. The structure was divided into subregions, namely, A, B and C as indicated in Figure 1. This was due to the complexity of analysing the non uniform geometry of the structure.

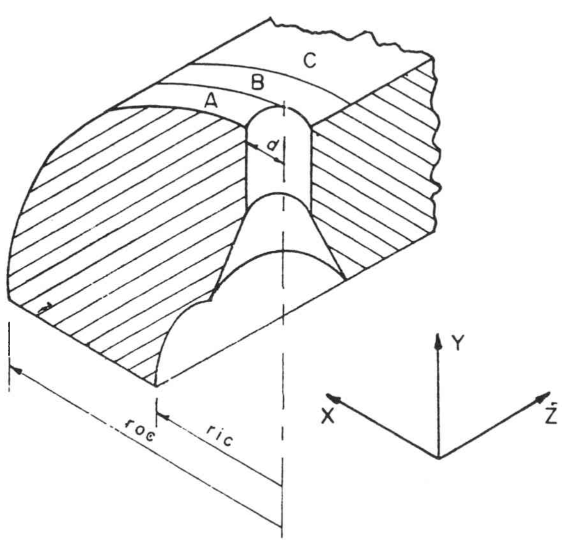

Figure 1 Chamfered cross-bored cylinder subregions

The stress concentration factor in this study was defined as the ratio of the maximum hoop stress to the hoop stress far away from the cross bore effects, i.e. nominal hoop stress. The finite element modelling of the structure adopted in this study was based on linear displacement approximation. In this case, the state of stress within the element is constant. The computer code developed here provided the state of averaged stresses at nodes. Stresses were evaluated at all Gauss points and averaged to get the average element stresses, i.e. the average of stresses computed for those elements which share nodal points. In this analysis, four types of isoparametric elements were used:

(a) wedge-type 1; (15 node and inside the structure),

(b) wedge-type 2; (15 node and on the cross bore surface),

(c) brick-type 2; (20 node and inside the material),

(d) pyramid-type 4; (13 node and along the chamfer knuckle).

Within any given subregion, the number of elements curved out was determined by the degree of accuracy required and the computer time required to solve the resultant problem.

Results and discussion

In the present study, a thick walled cylinder with cross bores of varying radii and a thickness ratio of 2 was considered. The hoop stress distribution in the cylinder is discussed for a cross bore radius variation of $0.5 - 2$ mm. These cross-sections where the surface nodal stresses are analysed are shown in Figure 2. The resulting hoop and stress concentration factor profiles are discussed below.

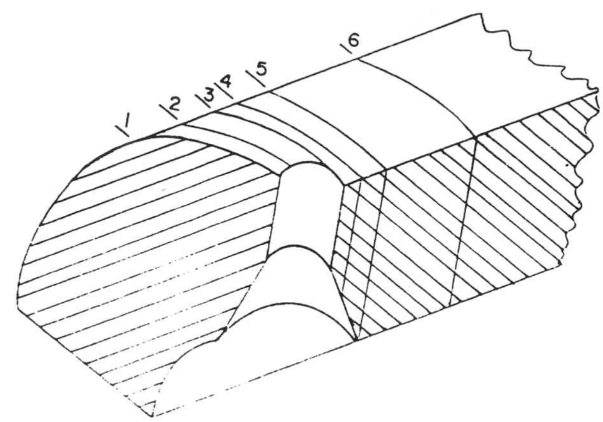

Figure 2 Cross-sections along the z-axis

(a) Hoop stress profiles

For reasons explained in (b), the 1 mm radius cross bore with a 2 mm long chamfer showed minimum stress concentration factor. The stress profiles discussed refer to the same cylinder analysed for the stress concentration factors. The chamfer angle was progressively increased from 15° to 57.5°. Curves 1 to 6 referred to in Figures 3 to 5 correspond to the cross sections 1 to 6 cut out in Figure 2. The corresponding hoop stresses on the loaded nodes of the main cylinder bore and the cross bore are discussed.

From the profiles presented, the hoop stress variations follow very similar patterns. The steady or the stress far from the cross bore is not the same for all the curves. This variation can be attributed to the degree of mesh refinement. However, this difference is quite small and of the order of 3.14%. This variation is dependent on the degree of discretization of the structure in the region. As the z-component of any point of interest increases, the value of the vertical distance where geometric discontinuity starts, increases. Hence, the peak stress, being the maximum hoop stress for any curve, shifts to the right for subsequent curves since the peak stress for each cross-sectional curve is observed to correspond to the geometric discontinuity on the inner radius of the cylinder. The hoop stress profiles are observed to cluster together till the chamfer edge is approached at a distance of approximately 12.5 mm away from the main bore centre. The curves show increase in hoop stress at different rates. The curves then attain different peak stresses with curve 5 reaching the highest for all the cases.

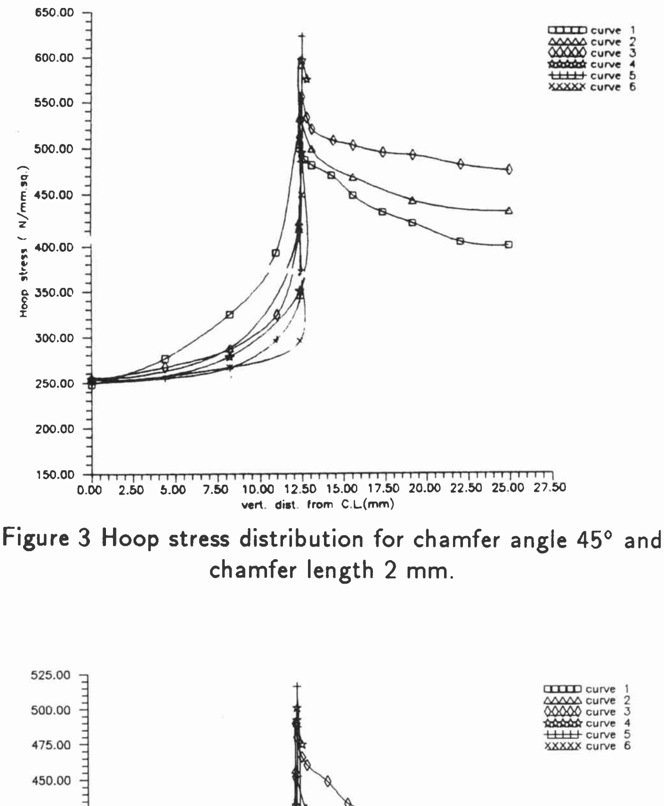

Figure 3 Hoop stress distribution for chamfer angle 45° and chamfer length 2 mm.

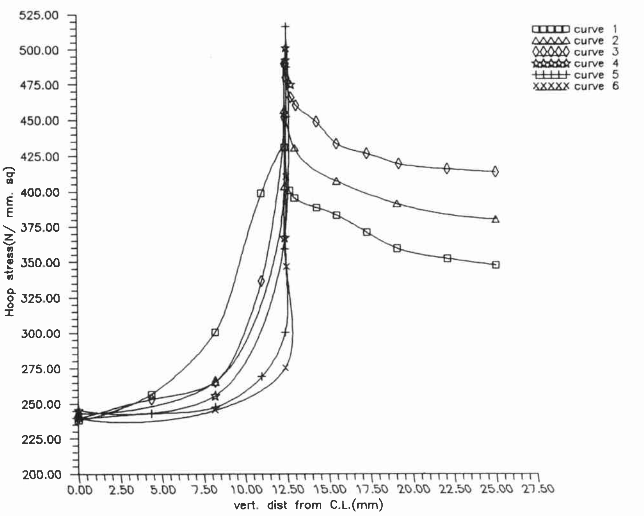

Figure 4 Hoop stress distribution for shamfer angle 50° and chamfer length 2 mm.

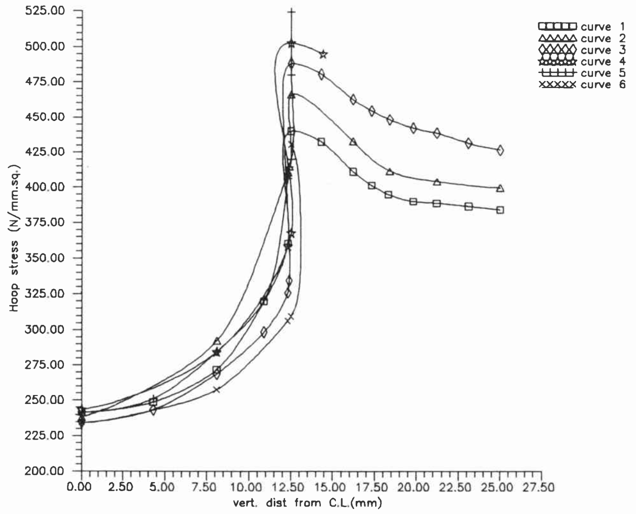

Figure 5 Hoop stress distribution for chamfer angle 52.5° and chamfer length 2 mm.

Generally, only the peak stress for each curve varies from geometry to geometry but the patterns are very similar. Increasing the chamfer angle, for a fixed chamfer length, shifts the geometric discontinuity to the left but this is hard to notice from the hoop stress profiles in Figures 3 to 5. Tables 1 and 2 summarize the maximum hoop stresses for the different geometries. These maxima were observed to occur at the chamfer to main bore intersection in the longitudinal section. These maxima values are of significance since an optimum geometry that minimizes the maximum hoop stress is normally selected in design.

Table 1 Maximum hoop stress values for a chamfer length of 2 mm

Chamfer angle (mm)	Max. hoop stress (N/mm²)
15	591.1
20	632.2
25	611.8
30	622.4
35	607.6
40	595
45	625
50	516.6
52.5	524
55	573.5
57.5	609.5

For the geometry of 2 mm chamfer length, 50° chamfer angle and 1 mm radius cross bore, the hoop stress values on the chamfer knuckle and the plain section of the cross bore are shown in Figure 6. From this figure, it is clear that the most stressed region is at the chamfer to main bore intersection with a hoop stress of 516.5 MPa. This stress distribution scenario is common.[13] It is clear that the stresses around the cross bore surface are much higher than elsewhere. The maximum hoop stress for each particular geometry has also been established and its location identified. From the plots, it can be deduced that the region in a cross-bored thick cylinder where failure would start is around the cross bore. This observation is important because measures have to be taken in the design and fabrication stages to strengthen this region by padding and reinforcing.

Table 2 Maximum hoop stress values for a chamfer angle of 50°

Chamfer length (mm)	Max. hoop stress (N/mm²)
0.5	615.7
0.75	550
1	660
2	516.5
3	656.6
4	644
5	698.4

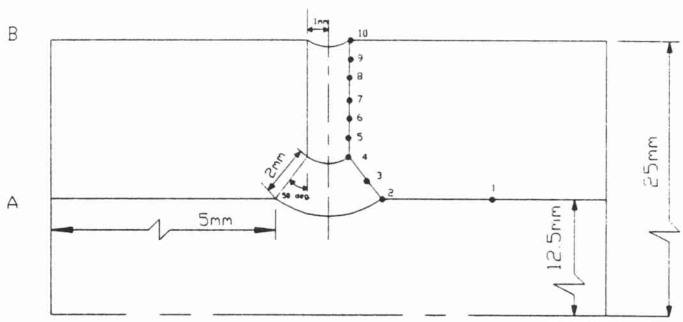

Figure 6 Meridional section hoop stresses for chamfered cross bore

Point No.	Hoop stress (N/mm²)
1	410.4
2	516.5
3	475.0
4	460.2
5	448.7
6	433.4
7	426.5
8	419.3
9	415.7
10	413.5

These observations contrast the work of some authors [13] and [4] who found the region of maximum hoop stress to be at the chamfer to cross bore intersection in the longitudinal section. This difference could be attributed to the number of elements used by these authors. For example, Friedman & Jones,[4] treating the thick cylinder as a thin shell, modelled the structure with 1 668 elements and 8 660 nodes while, in the present study, 68 elements were used with one element at the chamfer knuckle. However, the results of this work have been shown to be quite in conformity with other researches, e.g. Masu,[13] and Gerdeen.[6] From the stress profiles, it is not possible to identify the geometry with optimum hoop stress distributions. This is actually strictly not necessary because the peak stress is the value of prime interest. The fact that hoop stresses far from the cross bore attain values that are close, within 12.1%, to theoretical values confirms that the method used in this work is acceptable.

(b) Strength concentration factor profiles

For each particular geometry, the stress concentration factors were evaluated as the cross-bore radius was varied from 0.5 – 2 mm. For chamfer length variation of 0.5 – 5mm, variation of the chamfer angle depended on the particular chamfer length as well as the appearance of a minimum stationary point when the chamfer angle was plotted against the stress concentration factor. This determination of maximum chamfer angle was necessary in order to minimize the cases considered and to reduce computer running costs.

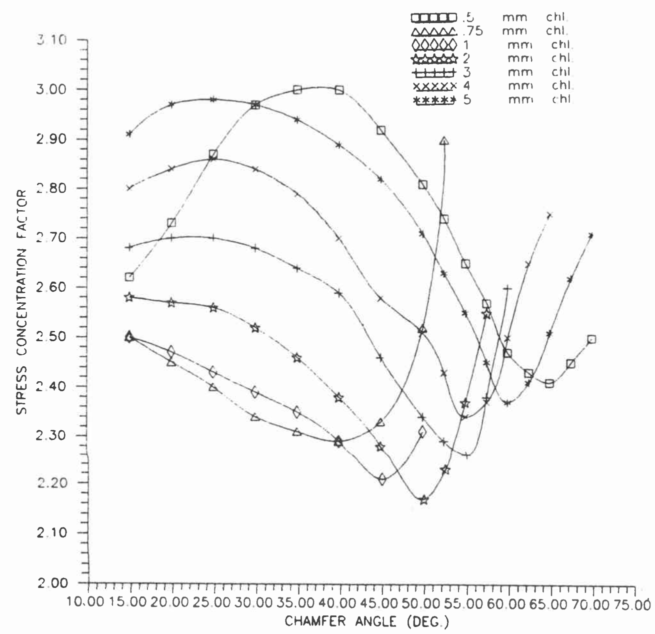

Figure 7 Stress concentration for a cross bore of 1 mm radius for varying chamfer lengths

The stress concentration factor results for the various cross-bore radii are represented together in Figure 7 due to similarity. For any geometry, the curves for chamfer lengths greater than 0.75 mm have similar patterns. As the chamfer angle increases, the stress concentration factor is minimally increased followed by a maximum. Any further increase in chamfer angle is followed by a decrease of the stress concentration factor till a minimum stress concentration factor value is attained. This minimum is followed by a sharp increase in the stress concentration factor. The maximum stress concentration factor for the various chamfer lengths is attained at different chamfer angles. For small chamfer angles, large chamfer lengths show higher stress concentration factor but for large chamfer angles, the order is reversed. The curve of 0.5 mm chamfer length shows a sharp rise in stress concentration factor as the chamfer angle is increased. The pattern thereafter is similar to other curves. The 2 mm chamfer length curve always has the minimum stress concentration factor at 50°. For the 0.5 mm cross-bore radius, the minimum stress concentration factor is 2.25. For the 1 mm cross-bore radius, the minimum stress concentration factor is 2.17. For the 1.5 mm cross-bore radius, the minimum stress concentration factor is 2.31. For the 2 mm cross-bore radius, the minimum stress concentration factor is 2.54. When the chamfer angle is held constant and the chamfer length varied, the stress concentration factor decreases exponentially till a minimum is attained. Further increase in chamfer length is followed by a gradual rise in stress concentration factor. An optimum geometry of 2 mm chamfer length, 1 mm cross-bore radius is observed for a thick cylinder with a thickness ratio of 2. For each chamfer length, the point of minimum stress concentration factor is of special interest for design purposes.

For a thick cylinder of thickness ratio of 2, the minimum stress concentration factor obtained was 2.17 and this corresponded to a cross-bore radius of 1 mm and a chamfer angle of 50°. The K_s ratio for this case is 12.5. This value of K_s is seen to be the best in minimizing the stress concentration factor for a cylinder of this geometry. For this reason, the stress profiles around the cross-bore region for the cross-bore radius of 1 mm were investigated and discussed as indicated in the previous section. It is clear that a cross bore with 1 mm radius is the most suitable given the other cylindrical geometric properties.

(c) Plain cross-bored cylinder

In order to investigate and ascertain the advantages of a chamfer in a cross bore, plain cross-bore geometry was investigated. The stress concentration factor obtained for plain cross-bored cylinders of various radii were established. For a cross bore of 0.5 mm radius, a stress concentration factor of 3.45 was obtained. For a cross bore of 1 mm radius, a stress concentration factor of 3.3 was obtained. For a cross bore of 1.5 mm radius, a stress concentration factor of 3.57 was obtained. For a cross bore of 2 mm radius, a stress concentration factor of 4.19 was obtained. The 1 mm cross-bore radius showed the minimum stress concentration factor of 3.3. A maximum peak stress of 808.5 MPa was seen to occur at the cross bore to the main bore intersection in the longitudinal section. The significance of this location is that being the most highly stressed region, any failure in the structure due to the internal pressure loading under consideration would most likely begin here. It is also noteworthy that for plain cross bores investigated, the 1 mm radius cross bore is the best design choice for the cylinder under investigation. It is observed that in using a cross bore of 1 mm radius, a 34.2% reduction in stress concentration factor is achieved.

Previous researchers working on cylinders of thickness ratio of 2 have obtained different values of stress concentration factor. Gerdeen [6] used a photoelastic technique and obtained a stress concentration factor of 3.32. Tan [14] and Fenner & Nadiri [3] used the boundary integral equation and obtained stress concentration factors of 3 and 3.7, respectively. Faupel & Harris [2] used the strain gauge and photoelastic methods and obtained a stress concentration factor of 3.02. Masu [13] used the finite element method and obtained a stress concentration factor of 3.03. In the present study, a stress concentration factor of 3.3 has been obtained. The above cited findings compare well with those obtained in the current work.

(d) Round cylinder

In order to establish the validity of the finite element algorithm used here, a test cylinder model without a cross bore was analysed.

The results of internal hoop stress of 234.3 MPa approaches the value of 266.7 MPa obtained from Lame's analytical procedure. This value with a 12.1% error is acceptable considering that the analytical solution should be approached if the number of elements considered is very high. In this work, 60 elements were used in the plain cylinder analysis.

(e) General discussion

From (b) and (c), it was observed that the introduction of chamfers to thick cylinders reduces the stress concentration factors. A reduction of the stress concentration factor is very important. Smaller safety factors may be used and hence less material is used in the forging and fabrication processes. This saving means that the heat treatments of the vessel material are less expensive, less labour is required and welding process is cheaper. This saving is appreciated because large ingots are used in the forging process and hence a small percentage in saving means a lot in terms of the weight and cost of steel used.

Conclusions

The aim of this study was to establish the hoop stresses and the stress concentration factors in a thick-walled cylinder with chamfered cross bores under internal pressure. These results were to be compared with those of a similar thick cylinder with a plain cross bore. The following conclusions can be made from this study:

(a) The hoop stress profiles which have been described above show that the hoop stress distributions and the maximum hoop stress in a thick cylinder of a given thickness ratio depends on the cross-bore radius, the chamfer angle and the chamfer length. It is noteworthy that in this work, the analysis sought to find for each geometrical combination, the maximum hoop stress and its location. This has been discussed in results and discussion. However, for design purposes, the least value of the maximum values is what the designer seeks.

(b) The introduction of a chamfer in a cross bore of a thick cylinder has been observed to reduce stress concentration factor. The amount of this reduction depends on the cross-bore radius chosen. At the optimum geometry of 1 mm cross-bore radius for a thick cylinder with thickness ratio of 2, a 34.2% reduction in the maximum stress concentration factor was observed. The percentage reduction is dependent on the thickness ratio of the cylinder, the cross-bore radius, the chamfer length and the chamfer angle.

(c) The hoop stress profiles indicate that the cross bore is a region of high stress concentrations. In addition, it has been established that around the cross bore, the chamfer to main bore intersection in the longitudinal section is the most highly stressed region.

(d) For the plain cross-bore cylinder, the minimum stress concentration factor was found to be 3.3 and this compared very favourably with the work of previous researchers.

(e) The stress profiles obtained can form a good guide for other researchers who may wish to consider other loading conditions, end constraints, plastic deformation and crack initiation in pressure vessels.

Recommendations

A number of areas need to be studied in order to make the study of the stress profiles around the cross bores more elaborate:

(a) Evaluation of the stress concentration factor for cylinders of varying diameter ratio and cross-bore radius.

(b) Consideration of the elasto-plastic cylinder case.

(c) Confirmation of the results by experimental methods such as photoelastic analysis and strain gauge methods.

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