Adaptive Control Strategy to Steer and Drive an Autonomous 4ws4wd Ground Vehicle

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Abstract - Autonomous vehicles are equipped with an increasing number of actuators to actively control the longitudinal and lateral dynamics of the vehicle. Amidst difference in terrains and roads that an autonomous vehicle drives through, precise coordination of the available actuators and effectors is needed to ensure an Autonomous Ground Vehicle (AGV) has improved manoeuvrability, accuracy and controllability. The increased number of actuators in a four-wheel steer and four-wheel drive (4WS4WD) architecture renders the control of the vehicle a challenge as the system becomes highly nonlinear. The control of an over-actuated, nonlinear and highly coupled system demands a superior control strategy. The controller should steer and drive the AGV to accurately track a path. The research proposes an adaptive MPC uses a linearized and a plant model updated at every instant. Preliminary results of the controller show that it bears potential to guide the 4WS4WD AGV to track a pre-defined path with minimal errors.

Key Words- Adaptive MPC, Autonomous Ground Vehicle, Control, Drive, Steer.

I. INTRODUCTION

MODEL Predictive Controller (MPC) is best suited for multiple-input and multiple-output system such as the AGV. Moreover, the predictive nature of the control approach enhances the computation of the control command hence accurate drive and steer of the vehicle. The AGV model is highly nonlinear which demands the use of a nonlinear MPC. However, the complexity of designing the nonlinear MPC and its computational load sets it aside compared to using an adaptive MPC. The adaptive MPC uses an updated and a linearized plant model at every instant to solve an optimization problem. The approach is superior due to the concept that the AGV model is updated at every time step hence bear the potential to generate an accurate control command.

II. SYSTEM MODEL

This section covers the derivation of the model that represent the vehicle dynamics.

A. *OVERVIEW*

The dynamics of motion for an AGV can be described using the well-developed Newton-Euler formalism. Considering input u and state ψ, the systems can be expressed by

$$\dot{\psi} = f(\psi, u) \tag{1}$$

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The AGV architecture brings together submodules that make up a vehicle include vehicle body dynamic model, driving unit dynamic model and tire model [1]. Based on SAE j670e [2], SAE j3016 [3] and ISO 8855 [2] vehicle dynamics terminology and guidelines are used in the derivation of the subsystems considered in the next section.

B. *EQUATIONS OF MOTION*

Four-wheel notation for the body dynamic in the driving task is governed by

$$\dot{\psi}(t) = f^{4w}\left(\psi(t), u(t)\right) \tag{2}$$

where $\psi(t) \in \mathbb{R}^n$ is the state of the system and $u(t) \in \mathbb{R}^{m_r}$ is the input, $n = 4$ is the number of states and $m_r = 6$ is the number of inputs. The state space representation is the system under study is expressed by

$$\dot{\psi}_1 = \left(-\left(2Cf + 2Cr\right)/mVx\right)\psi_1 + \left(-Vx - \left(G\right)/mVx\right)\psi_3 + (2Cf/m)\delta$$

$$\dot{\psi}_2 = \psi_3$$

$$\dot{\psi}_3 = \left(-\left(G\right)/Iz/Vx\right)\psi_1 + \left(-Vx - \left(2Cflf^2 - 2Crlr^2\right)/mVx\right)\psi_3 + (2Cflf/Iz)\delta$$

$$\dot{\psi}_4 = \psi_1 + \left(Vx\right)\psi_2 \tag{3}$$

where $\psi_1, \psi_2, \psi_3, \psi_4$ represents lateral position, lateral velocity, yaw angle and yaw rate and G is $2C_f l_f - 2C_r l_r$.

The state space representation of the system based on both dynamic and kinematic equation is summarized into

$$Pos = \begin{bmatrix} x & y & \theta \end{bmatrix}^T \tag{4}$$

$$Vel = \begin{bmatrix} V_{lng} & V_{rdl} & V_{ang} \end{bmatrix}^T \tag{5}$$

$$Acc = \begin{bmatrix} a_{lng} & a_{rdl} & a_{ang} \end{bmatrix}^T \tag{6}$$

where $a_{lng}, a_{rdl}, a_{ang}$ represent longitudinal, radial and angular acceleration, $V_{lng}, V_{rdl}, V_{ang}$ represent longitudinal, radial and angular acceleration and x, y, θ represents longitudinal and lateral positions and yaw angle respectively.

III. AGV CONTROLLER DESIGN

This section informs system states, input, outputs, problem formulation, control policy and control algorithm.

A. *STATES, INPUTS AND OUTPUTS*

From the vehicle model designed above, it is notable that the vehicle is coupled and highly nonlinear. The vehicle takes 6 inputs i.e. 4 torque inputs (T_1, T_2, T_3 and T_4) and 2 steering angle inputs (δ_f and δ_r). Considering vehicle motion as the output, three acceleration measurement accomplish the output variables (6).

In view of the 3 outputs and 6 inputs, the design is rendered an over-actuated system which complicates maintaining accuracy and robustness. Wheel slip leads to lateral and longitudinal forces at individual wheels and is determined by kinematic states (5).

In a real-life scenario, the AGV's four wheels are subjected to varied external disturbances as ground forces. Independent force control for the AGV's four wheels is a solution that enhances dynamic states stability [4]. Moreover, a superior controller design enhances manoeuvrability and stability without regard to complex terrain conditions. The adaptive MPC design considers the system of equation (3-5) represented by

$$\dot{\psi}(t) = A(t)\psi(t) + B(t)u(t) \tag{7}$$

and output governed by

$$y(t) = H(\psi(t), f(t)) \tag{8}$$

The system (7) is discretized so as to form discrete state-vector defined by

$$\psi_d(k+1) = A_d\psi_d(k) + B_d u_d(k) \tag{9}$$

$$y_d(k) = H_d\psi_d(k) \tag{10}$$

B. PROBLEM FORMULATION

In controlling an AGV, the path planner generates reference paths and a velocity profile. The parameters then serve as a reference for the control problem. The need to steer and drive an e-vehicle to track the reference paths and velocity, known as a servo system, needs a controller that can achieve the set path and velocity within the shortest time, with minimal errors while ensuring system stability. The AGV problem under study is a tracking problem and is well expressed using an error model (13). Controlling an AGV involves regulating deviations from steady-state trajectory from the road curvature and velocity profile.

Remark 1: *the system of matrix* (A_d, B_d) *in* (9) *is controllable and there exists a feedback stabilizing matrix of gain* K.

$$\psi_d(k+1) = (A_d - BK)\psi_d(k) + B_d u_d(k) \tag{11}$$

The gain matrix K system can thus be obtained based on well documented approaches [5]–[8] such as pole placement. The approach of remark 1 are considered in the control problem of a system with an infinite response. Due to the limitation in computation time, infinite suboptimal solutions that repeatedly solve a finite time optimal control problem are handy.

Remark 2: *with consideration of system(9), remark 1 and time constraints, a feedback control law* Δu *asymptotically stabilizes a closed-loop system.*

$$\Delta u(\Delta\psi) = -K\Delta\psi + v^* \tag{12}$$

With the computation of control input $\Delta u(k)$ that determine system trajectory $\psi(k)$while satisfying design requirement, an auxiliary control command v^*can be determined as the result of an optimal control problem. Optimal control problem can effectively be solved through [9]–[11].

Adaptation allows a recursive and refined vehicle model estimate which ultimately offer comfort and safety. Considering the nonlinear system expressed by (3-5), steering control was considered by Bujarbaruah et al.[8] where a steering offset estimation was defined. The steady-state trajectory error model as

$$\Delta\dot{\psi}_t = A\Delta\psi_t + B_1\Delta\delta_t + B_2\Delta\rho_t + E\theta_a + w_t \tag{13}$$

where$\Delta\psi, t \geq 0$ represents the realized error trajectory for a closed-loop system and w_t takes care of unmeasured disturbance to the system that is of an additive nature.

Remark 3: *with the knowledge of* θ_a, *the domain* Θ *regarded as the feasible parameter set, can be defined*.

$$\begin{aligned}\Theta_t &= \Delta\dot{\psi}_t - A\Delta\psi_{t-1} - \ldots \\ &\ldots - B_1\Delta\delta_{t-1} - B_2\Delta\rho_{t-1} - E\theta_t\end{aligned} \tag{14}$$

and

$$\Theta_t \in \mathsf{W},\ t \geq 0 \tag{15}$$

$$\theta_t \in \mathsf{R}^p : \tag{16}$$

where the parameter $\theta_a$is composed of the steering offset θ_δ and driving offsetθ_ρ. In that regard, the parameter set is estimated from other previous vehicle data. The *feasible parameter set* at time t undergoes an update every time step as data on the input-output state is gathered. The steady state trajectory and inputs reveals the ability to progressively update the knowledge of Θ with consideration of all other information from previous time instants.

C. CONTROL POLICY

From remark 1 and 2, the control law to steer and drive the AGV of system (8-9) can thus be design considering an affine feedback policy $\pi(.)$ for control approximation.

$$\pi_t(.) : \Delta\delta_t(\Delta\psi_t) = -K\Delta\psi_t + v_t^* \tag{17}$$

$$\pi_t(.) : \Delta\rho_t(\Delta\psi_t) = -K\Delta\psi_t + v_t^* \tag{18}$$

where $K \in \mathbb{R}^{m\times n}$ is the fixed stabilizing state feedback gain chosen by Bujarbaruahet al. [8] as optimal LQR while the auxiliary control v_t is computed from (20) as directed by [11], [12]. Constraints are then imposed in the form (13), (19),(21) and (22) noting that the control objective is to keep $\Delta\psi_t$ small.

$$C\Delta_t\psi + D_1\Delta\delta_t + D_2\Delta\rho_t \leq b \quad (19)$$

The control problem computed in finite time is depicted by solving a finite receding horizon optimal control problem based on squared Euclidean norm consistent with guide by [11]

$$\min \sum_{i=0}^{Hp} (\Delta\psi_i)^T Q(\Delta\psi_i) + \ldots\ldots$$
$$\ldots\ldots + \sum_{i=0}^{Hp-1} [(\Delta v_i)^T R(\Delta v_i) + v_i^T R v_i] \quad (20)$$

s.t
(13)
(19)
$g(\psi) = 0$ (21)
$h(\psi) \leq 0$ (22)

where H_p is the desired prediction horizon that then determines the control horizon. With the consideration that the system (13) is updated through criteria (14), thus computation of control law by conventional MPC consistent with (20) is dependent on a model that is updated at every sampling time. This renders adaptive MPC able to generate a current control command based on a true model representation[13]–[15].

D. ADAPTIVE MPC ALGORITHM

Bujarbaruahet al. [8] showed how an MPC is able to achieve a recursively feasible control policy. The adaptive control algorithm considered is;

1. **While** $\theta_a = constant$ **do**
2. Obtain road curvature $C(s)$ and velocity $V_{lngref}(s)$. Compute corresponding steady state trajectory $\psi_{ss}(s)$ drive force input $\rho_{ss}(s)$ and steering angle input $\delta_{ss}(s)$. Set $t = 0$; initialize *feasible parameter set* Θ_0
3. **While** $V_{lngref}(s)$ and $C(s)$ unchanged w.r.t step 2 **do**
4. Compute the terminal invariant set X_t^N. Compute v_t^* from (20) and simultaneously apply steering command
$$\delta_t = \delta_{ss}(s) - K(\psi_t - \psi_{ss}(s)) + v_t^* \quad (23)$$
And drive command
$$\rho_t = \rho_{ss}(s) - K(\psi_t - \psi_{ss}(s)) + v_t^* \quad (24)$$
5. Update $\Theta_{(t+1)}$ using (14). Set $t = t + 1$
6. **End while**
7. Set $\Theta_0 = \Theta_t$. return to step 2
8. **End while**

E. MPC DESIGN PARAMETERS

The selection of MPC parameters affects controller performance and the computation complexity. The design of sample time, prediction horizon, control horizon, constraints and optimization weights informs the process.

Computationally, an MPC algorithm ought to solve an online optimization problem at every time step. The controller **sample time** determines the rate of executing the control algorithm. Intuitively, by selecting a big sample time, controller will not react fast enough. By selecting a small sample time, controller reacts faster to disturbances and set-point changes. While causing an excessive computational load. Sample time should in that regard, fit $10 - 20$ samples within the rise time of the open-loop system response. Vehicles should have a response time of $0.83\ s$ as stated by [16]. Based on the sampling time criteria above, the AGV should be sample at $T_s = \frac{0.83}{18} = 0.046\ s = 46\ ms$.

The MPC **prediction horizon** shows how far the controller predicts into the future and be able to cover the AGV dynamics. The Prediction horizon should be within 20-30 samples of the system response. As established above, system response at $0.83\ s$ means the design of the AGV prediction horizon is within $\frac{0.83}{25} = 0.033\ s = 33\ ms$.

The **control horizon** informs the time into the future that the control action can be predicted. Since instantaneous control achieves better control of the system, rule of thumb requires that the control horizon be within $10\% - 30\%$ of prediction horizon i.e. $33 \times 0.15 = 4.95\ ms$

From design specifications as presented above, and during simulation, it was notable that adjusting sampling time to $0.1s$ led to better control than $0.83s$ as shown in Fig. in section IV. Moreover, adjustment to prediction horizon to 15 and control horizon to 5 yielded better results.

Constraints should include actuation saturation, MV rates constraints and manipulated variables (drive torque T and steering angle δ). The associated input constraints in table 1 cannot be relaxed as they are physical necessities and may put system in jeopardy. On the other hand, state and output constraints can be relaxed as they are merely operational desirables. The design used output weights as $[2.8292\ \ 0.2829]$. **Weights** ratio between output weights and input and input rates.

For the finite time, discrete model (discretization time $t_d = 100\ ms$). Linearized system state equation (29) and computation of an optimal control as in (20)

s.t

$$-30 \leq \delta \leq 30 \quad (25)$$
$$-2500 \leq F_d \leq 2500 \quad (26)$$
$$\Delta\delta \leq 8 \quad (27)$$
$$\Delta F_d \leq 25 \quad (28)$$
$$\psi^+ = A\psi + Bu \quad (29)$$
$$e_{k+1} = f(e_k, \Delta u_k), k = 0, \ldots H_p - 1 \quad (30)$$

where H_p is the prediction horizon and the matrix Q is determined to be positive definite while R is chosen to be positive semi-definite to avoid introduction of offset in the controlled variable [17]. Noted that $e_k = y_k - y_{ref}$, value of y_{ref} is provided by the path planner and y_k is the measured output.

The adaptive MPC then implement $u(t) = u_0^*$ at every iteration.

Table 1 Input and rate constraints

I/O Variable	**max**	**min**	Δ
Steering angle, δ	30^0	-30^0	8^0
Drive forces, F_d	$2500\ N$	$-2500\ N$	$25\ N$

IV. SIMULATION RESULTS

The system used was a 5-seater salon e-vehicle whose parameters are presented in table 2.

Table 2 e-vehicle Parameters

Parameter	**Quantity**	**Units**
L_f	2.1	m
L_r	2.2	m
c	1.8	m
h	1.5	m
$Mass, m$	1036	kg
Wheel radius	0.24	m

Holding longitudinal velocity constant and testing how the system lateral position and yaw angle trajectory evolves against reference is shown in Fig 3 and Fig 4 respectively. For velocity of 15m/s the steady state lateral position error was $e_\theta = 4.014 - 4.011$, $e_\theta = 0.003\ m$.

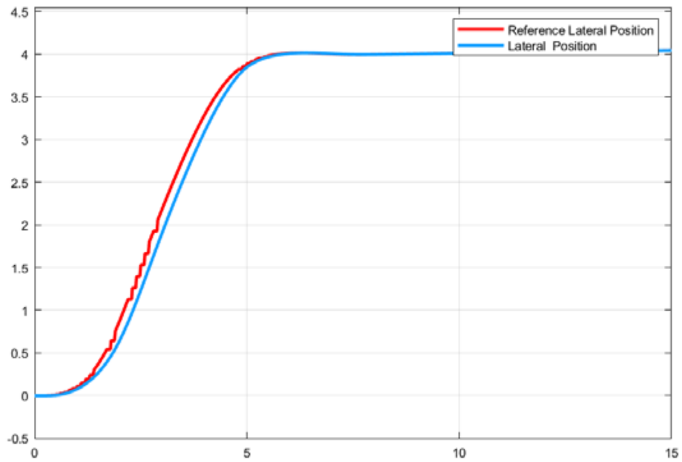

Fig. 1 Lateral position trajectory of the vehicle

Considering the yaw angle reference and the trajectory taken by the vehicle, a yaw error of $0.0014 - 0.0010 = 0.0004$ which is an error of0.23^0.

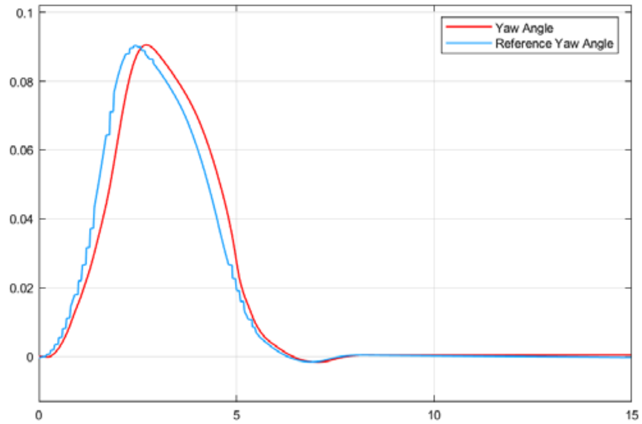

Fig. 2 Yaw angle trajectory of the vehicle

The system under consideration was subjected to sudden slip in its course and the recovery of the vehicle back to its track is shown in Fig. 5 and Fig. 6. At $4\,s$ the vehicle undergoes a slip that results in a maximum lateral error of 2.7 m. In response to the slip that occurs, the vehicle is steered back and recovers to its right path within $2s$.

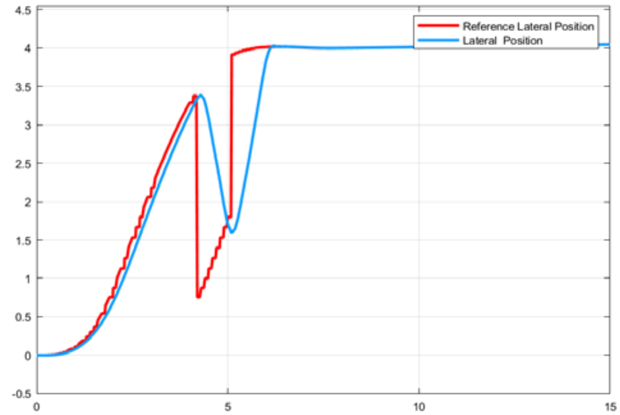

Fig. 3 Trajectory of vehicle recovery after slip

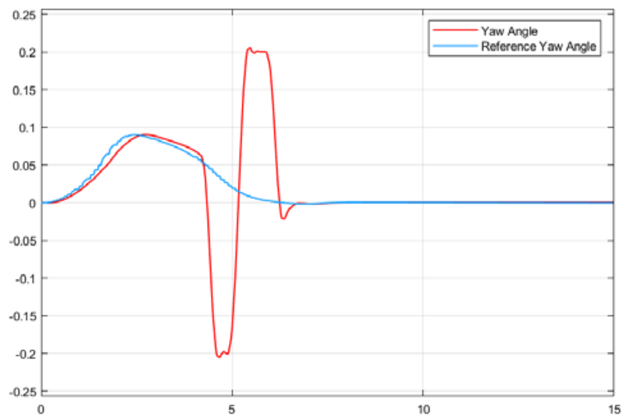

Fig. 4 Yaw angle response to vehicle slip

V. CONCLUSION

Results of the adaptive MPC in the steering of the AGV shows improvement in tracking the reference path. This is however achieved while holding a constant longitudinal velocity. The variations of control horizon in the simulation is better at a scaling of 30% in comparison to the prediction horizon. The sampling times for the adaptive section is a critical parameter to meet to avoid deteriorating MPC performance. Future work would involve introducing variation in drive velocity according to velocity reference. In addition, varied road terrains (cornering and tire stiffness) will be varied and the performance of the adaptive MPC assessed.

VI. REFERENCES

[1] Q. Tan, P. Dai, Z. Zhang, and J. Katupitiya, “MPC and PSO Based Control Methodology for Path Tracking of 4WS4WD Vehicles,” *Appl. Sci.*, vol. 8, no. 6, p. 1000, 2018.

[2] S. A. E. International, “(R) Vehicle dynamics terminology,” 2008.

[3] SAI International, “J3016: Taxonomy and Definitions for Terms Related to On-Road Motor Vehicle Automated Driving Systems - SAE International,” *SAE International*, 2014. .

[4] J. Kong, M. Pfeiffer, G. Schildbach, and F. Borrelli, “Kinematic and dynamic vehicle models for autonomous driving control design,” in *IEEE Intelligent Vehicles Symposium, Proceedings*, 2015.

[5] S. Skogestad and I. Postlethwaite, “Multivariable Feedback Control???Analysis and Design,” *IEEE Control Systems*. 2007.

[6] V. G. Boltyanski and A. S. Poznyak, “Linear Quadratic Optimal Control,” in *Systems and Control: Foundations and Applications*, 2012.

[7] L. Keviczky, R. Bars, J. Hetthéssy, and C. Bányász, “State feedback control,” in *Advanced Textbooks in Control and Signal Processing*, 2019.

[8] M. Bujarbaruah, X. Zhang, H. E. Tseng, and F. Borrelli, “Adaptive MPC for Autonomous Lane Keeping,” no. May, 2018.

[9] A. Ferramosca, D. Limon, I. Alvarado, T. Alamo, and E. F. Camacho, “MPC for tracking with optimal closed-loop performance,” *Automatica*, 2009.

[10] T. Besselmann, J. Löfberg, and M. Morari, “Explicit MPC for LPV systems: Stability and optimality,” *IEEE Trans. Automat. Contr.*, 2012.

[11] F. Borrelli, A. Bemporad, and M. Morari, “Predictive control for linear and hybrid systems,” *Prep. Available Online Http//Www. Mpc. …*, 2015.

[12] D. Q. Mayne, J. B. Rawlings, C. V. Rao, and P. O. M. Scokaert, “Constrained model predictive control: Stability and optimality,” *Automatica*, 2000.

[13] V. Adetola, D. DeHaan, and M. Guay, “Adaptive model predictive control for constrained nonlinear systems,” *Syst. Control Lett.*, 2009.

[14] M. Lorenzen, F. Allgöwer, and M. Cannon, “Adaptive Model Predictive Control with Robust Constraint Satisfaction,” *IFAC-PapersOnLine*, 2017.

[15] E. Kural and B. A. Güvenç, “Model predictive adaptive cruise control,” in *Conference Proceedings - IEEE International Conference on Systems, Man and Cybernetics*, 2010.

[16] V. V. Dixit, S. Chand, and D. J. Nair, “Autonomous vehicles: Disengagements, accidents and reaction times,” *PLoS One*, 2016.

[17] K. R. Muske, “Steady-state target optimization in linear model predictive control,” in *American Control Conference*, 2002, pp. 3597–3601.