Battery and Motor Sizing for Electric Tricycles

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Abstract**—**In the electrification of bicycles, tricycles, wheelchairs, and other velocipedes, there is a need to size the battery and the motor according to the expected load. This is because oversizing these components leads to expensive, inefficient and underutilized systems whereas under sizing them makes the system unsafe due to the risk of overheating and component damage. This paper aims to address a procedural method for sizing the battery and motor for an electric tricycle. To achieve this objective, a working electric tricycle model was developed. By defining its top speed, acceleration time, and estimated mass, the required power output and rating of the motor was determined. Based on the voltage, power output and efficiency of the motor, the required battery size for the system was also determined. The battery capacity was chosen by considering the expected running time, load and acceleration time of the tricycle. In summary, the approach proposed uses the design parameters of mass, acceleration and velocity to determine the load-power requirement; since power is a function of force and velocity. The load requirement is subsequently used to size the components. Extrapolation of this method for other electric velocipedes is recommended.

Keywords— Battery, E-bikes, Electrification**,** Motor, Power.

I. Introduction

This paper was inspired by the Jibebe project, which involved the electrification of a tricycle for the handicapped that was commissioned by the Association of the Physically Disabled in Kenya (APDK). The type of motor to be used was the brushless DC (BLDC) motor [1]–[3] while the rechargeable battery to be used was a custom configuration of lithium-ion cells [4]–[6]. This is because lithium-ion cells have higher charge density as compared to the conventional lead-acid batteries[7]. While conducting research on how to size these components for the project, it was discovered that information on proper component sizing was not readily available online. This is the focus of this paper which is a consolidation of all the information that was found in addition to the authors' contribution with the intent of describing a procedure. To document this procedure, four major problems are systematically tackled with regards to the power requirements of the tricycle:

i. To obtain a solution for the power required for acceleration of the tricycle.
ii. To obtain a solution for the power required to overcome aerodynamic drag during motion.
iii. To obtain a solution for the power required to maintain traction during motion.
iv. To use the solutions obtained to size the motor and battery components.

The Jibebe tricycle will be used as a case study throughout this paper to demonstrate the applicability of the solutions. The design specifications are as follows:

i. The maximum velocity of the tricycle would be 16 km/h.
ii. The tricycle would be required to accelerate from rest to top speed in 9 s.
iii. The mass (*m*) of the tricycle was estimated as 150 kg.

The parameters of the tricycle are listed in Table I:

TABLE I: PARAMETERS FOR THE JIBEBE TRICYCLE

v [m/s]	a [m/s^2]	m [kg]	g [m/s^2]
4.4444	0.4938	150	9.81

II. Acceleration Power

The generic differential equations that describe the acceleration (*a*), velocity (*v*) and displacement (s) of the tricycle [8], [9] are given as:

$$\begin{cases} \frac{d^2 s}{dt^2} = A(t) \\ \frac{ds}{dt} = \int A(t)\,dt + s'(0) = V(t) \\ s = \int V(t)\,dt + s(0) \end{cases} \quad (1)$$

A. Constant Acceleration Design Model

We specify some conditions to simplify the generic equations above: Firstly, the tricycle has a constant acceleration and therefore exhibits linear velocity motion. That is, A(t) = a.

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$$\begin{cases} \frac{d^2s}{dt^2} = a \\ \frac{ds}{dt} = \int a\ dt + s'(0) = at + u \\ s = \int (at + u)\ dt + s(0) = \frac{1}{2}at^2 + ut + C \end{cases} \tag{2}$$

Where $s'(0) = u$ and $s(0) = C$

Secondly, the tricycle starts from rest implying zero initial velocity, and no initial displacement had occurred before time $t = 0$. In other words, the integration constants are zero.

$$\begin{cases} \frac{ds}{dt} = v = at \\ s = \frac{1}{2}at^2 \end{cases} \tag{3}$$

There are two approaches to solve for the required power: the kinetic energy approach and the kinematic equation approach.

For the kinetic energy approach, we begin with the definition that power, *P*, is the rate of doing work or transferring energy. The referenced energy in the case of motion of a body on a flat plane is kinetic energy. Therefore:

$$P = \frac{dE}{dt} = \frac{1}{2}m\frac{dv^2}{dt} \tag{4}$$

Substituting Eq. 3 in Eq. 4 and since the conditions in Eq. 2 assert that acceleration is a constant:

$$P = \frac{1}{2}ma^2\frac{dt^2}{dt} \tag{5}$$

Recall that *dt* is a 'small change in *t*' (Δt) and dt^2 is a 'small change in t^2' (Δt^2) as the changes in both cases become infinitesimal. Also, let t_2 be the final acceleration time and t_1 be the initial acceleration time:

$$\begin{cases} \Delta t = t_2 - t_1 \\ \Delta t^2 = {t_2}^2 - {t_1}^2 = (t_2 + t_1)(t_2 - t_1) \end{cases} \tag{6}$$

Substituting Eq. 6 in Eq. 5 yields:

$$P = \frac{1}{2}ma^2(t_2 + t_1) \tag{7}$$

Eq. 7 can be used to calculate power required to accelerate from initial velocity associated with t_1 to final velocity associated with t_2. Instantaneous acceleration power ($P_{inst.}$) can be obtained by letting the initial velocity (t_1) and the final velocity (t_2) be equal in Eq. 7.

$$P_{inst.} = ma^2t \tag{8}$$

Eq. 8 implies that the maximum instantaneous power occurs when *t* is maximum. Starting from rest, under constant acceleration, *t* is maximum when it is equal to the acceleration period (*T)*, hence:

$$P_{max} = ma^2T \tag{9}$$

To get the average acceleration power ($P_{ave.}$), we integrate eq. 8 with respect to time between the limits zero (rest) to the maximum value (*T*) and divide the result by the period of acceleration *T*.

$$P_{ave.} = ma^2\frac{T}{2} \tag{10}$$

It can be seen that for this model, the average power is half the maximum power.

For the kinematic equation approach, we begin with the definition that power is the product of force and the velocity resulting from the applied force. Additionally, from Newton's second law, force is equal to the rate of change of momentum (the product of mass and acceleration). Power therefore is given as:

$$P = m \cdot a \cdot v \tag{11}$$

Substituting Eq. 3 in Eq. 11 yields Eq. 8, which results in Eq. 9 and Eq. 10.

Case study

The Jibebe tricycle used the constant acceleration design model although it assumes that the rider throttles—or in other words accelerates—the tricycle at a constant rate. There are ways to constrain the acceleration of the tricycle to a constant value using microcontroller systems. However, even without employing a microcontroller to control the throttle rate, this model may be used to yield estimates of the expected power consumption. Using the parameters given in table I, the tricycle was found to draw a maximum power of 330 W and an average power of 165 W. The motor and battery are sized using the average power solution. A caveat needs to be given with regards to the conclusion above. The acceleration power never actually gets to 330 W but infinitely approaches the value as acceleration time, t, approaches 9 s. Instantaneous power is not a physical quantity; it gives information on the power limit of the tricycle as the acceleration time tends to a time t. Physical power values that describe power drawn between an initial time and a final time can be computed using Eq. 7. Fig. 1 shows a graph of instantaneous acceleration power versus time from which maximum acceleration power can be read at $t = 9s$ whereas fig 2. shows a graph of average acceleration power versus time. Note that $P_{ave.}$ is half the value of $P_{max.}$.

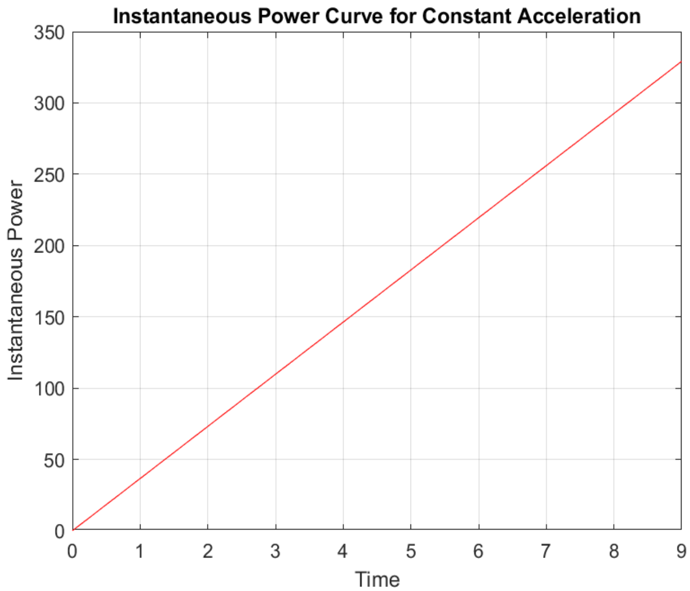

Fig. 1: Graph of instantaneous power against time for the Jibebe tricycle

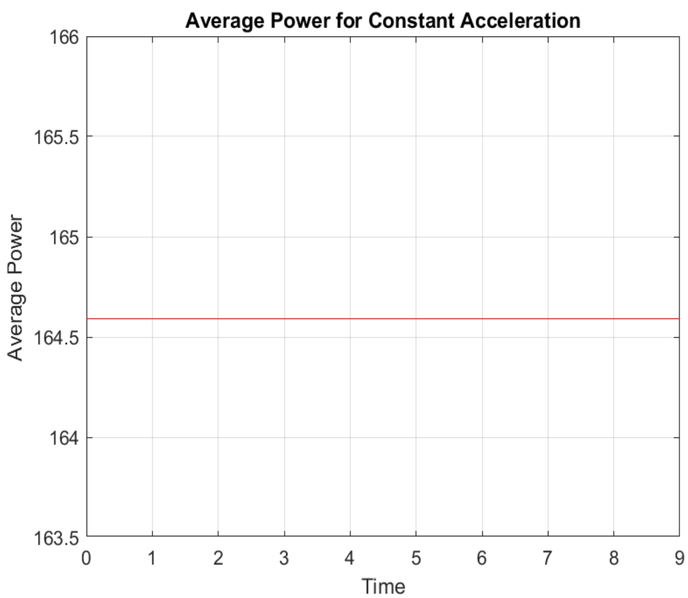

Fig. 2: Graph of average power against time for the Jibebe tricycle

B. Exponential Acceleration Model

There are instances where the velocity and acceleration of the tricycle follow an exponential curve. An exponential curve implies a negative exponent; otherwise, the quantities described would diverge. An example of such a model is in a case where the rate of change of velocity is controlled by an RC delay circuit. In such a case, the velocity (v) would be given as:

$$v = v_0(1 - e^{-kt}) \tag{12}$$

Where v_0 is the maximum velocity of the tricycle and k is an arbitrary constant. In the case of an RC circuit, k is the reciprocal of the circuit's time constant (τ). Acceleration is given as the derivative of Eq. 12 with respect to time:

$$a = \frac{dv}{dt} = kv_0 e^{-kt} \tag{13}$$

The displacement equation is not required in the discussion to follow but can be obtained by integrating Eq. 12 with respect to time:

For the kinetic energy approach, Eq. 4 is used. First obtain the square of Eq. 12 and replace it in Eq. 4. Then, perform the differentiation with respect to time to obtain the instantaneous power function ($P_{inst.}$):

$$P_{inst.} = mkv_0^2(e^{-kt} - e^{-2kt}) \tag{14}$$

To get the time (t) corresponding to the maximum power, we equate the derivative of Eq. 14 to zero.

$$\frac{dP_{inst.}}{dt} = mk^2v_0^2\left(2e^{-2kt} - e^{-kt}\right) = 0 \tag{15}$$

Solving for time corresponding to maximum power using Eq. 15 and replacing the solution in Eq. 14 yields the maximum power equation ($P_{max.}$) as:

$$P_{max.} = mkv_0^2(e^{\ln\left(\frac{1}{2}\right)} - e^{2\ln\left(\frac{1}{2}\right)}) \tag{16}$$

To get the average acceleration power, we integrate the instantaneous power with respect to time between the limits zero (rest) to the maximum acceleration time (T) and divide the value by the period of acceleration (T).

$$P_{ave.} = \frac{mkv_0^2}{T}\left(\frac{1}{2k}\left[e^{-2kT} - 1\right] - \frac{1}{k}\left[e^{-kT} - 1\right]\right) \tag{17}$$

The kinematic equation approach proceeds as defined in Eq. 11. That power is the product of mass, acceleration and velocity. Multiplying mass by Eq. 12 and Eq. 13 gives Eq. 14. As a corollary, Eq. 16 and Eq. 17 follow.

Case study

In the design phase of the Jibebe tricycle, consideration was made for the use of an RC delay circuit to control the acceleration rate. The value of k as previously mentioned would be related to the RC time constant of the delay circuit. k was obtained as 0.5556 after designing the RC circuit.

In this case, the maximum power was found to be 412 W and the average power was 162 W. Fig. 3 shows a graph of instantaneous acceleration power versus time under various k values from which maximum acceleration power can be read as the peak values whereas fig 4. shows a graph of average acceleration power versus time under various k values. Note that in both figures, the red plots (k = 0.5556) indicate the characteristics of the Jibebe tricycle. Notice from fig 3. that as k is increased, the maximum instantaneous power is increased and occurs earlier. If an RC circuit is used, k is $\frac{1}{\tau}$. The conclusion is thus that for smaller time constants the instantaneous peak value is larger and occurs faster. Additionally, for larger k (smaller τ), the instantaneous power decays faster affecting the acceleration time of the tricycle: for $k = 1$, $T = 6.5$ s; for $k = 1.5$, $T = 4.5$ s; and for $k = 2$, $T = 3.5$ s.

This asserts that *k* controls the tricycle acceleration time. Also, it can be seen in fig.4, that as *k* is increased, the average power increases up to a limit which can be precisely calculated by computing the limit as *k* approaches infinity of Eq. 17: the limit is 164.6058 W.

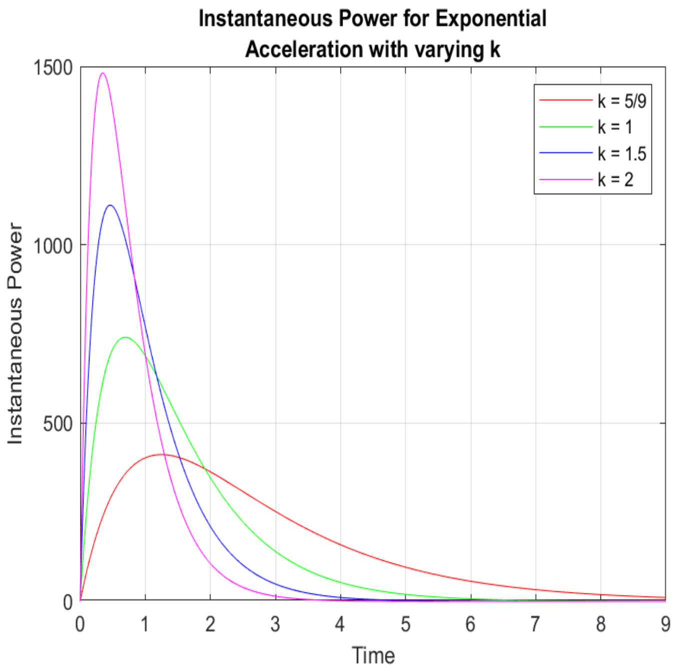

Fig. 3: Graph of instantaneous power against time for varying k

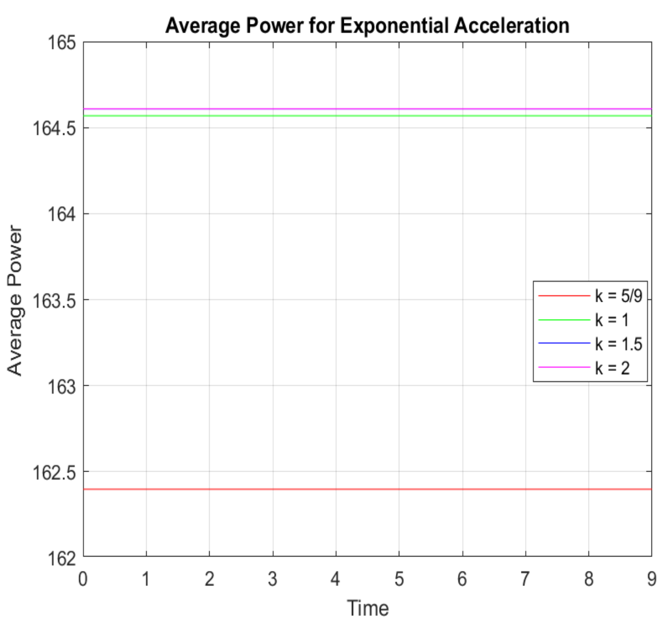

Fig. 4:Graph of average power against time for varying k

III. Aerodynamic Drag Power

Any object moving through a fluid will experience a drag: a net force in the direction of flow due to the pressure and shear forces on the surface of the object [10]. The aerodynamic drag is the force felt by a moving body due to the resistance of air. It is dependent upon the following factors: the density of air (ρ), the drag coefficient of the body (C_D) which is dependent on the shape of the body, the frontal area of the body (A) and the velocity of the body (v) [10]. The relationship is given by the formula below:

$$F_D = \frac{1}{2} \cdot \rho \cdot A \cdot C_D \cdot v^2 \quad (18)$$

From the aerodynamic drag force given in Eq. 18, the power required to overcome drag is obtained by taking the product of the drag force and the velocity of the tricycle.

$$P_D = \frac{1}{2} \cdot \rho \cdot A \cdot C_D \cdot v^3 \quad (19)$$

Case study

The density of air was taken at sea level because generally, as altitude decreases, then air pressure increases. The International Standard Atmosphere states the density of air as 1.225 kg/m^3 at sea level with the temperature being 15 ^{o}C [11]. The frontal area of the tricycle with a rider was measured as 1 m^2. The worst-case coefficient of drag for a bicycle when the rider is oriented upright was taken as 1.1 [10]. Therefore, the worst-case power required to overcome aerodynamic drag for the Jibebe tricycle was found to be 30 W.

IV. Traction Power

When rolling, a tire is deformed by the load exerted on it, flattening out on the contact patch, this repeated deformation causes energy loss known as rolling resistance or rolling friction [12], [13]. Tires of any vehicle such as a bicycle or a tricycle experience this force and are required to overcome it in order to maintain their momentum. This force that counters the rolling resistance of a vehicle is known as the traction force and its associated power is known as the traction power. A free body diagram of these forces is given below:

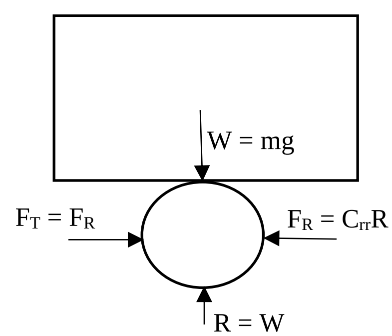

Fig. 5 Free body diagram illustrating traction

Where:
W = Weight/Normal force
R = Weight reaction force
F_R = rolling resistance or rolling friction
C_{rr} = rolling resistance coefficient
F_T = traction force

From the diagram, it can be seen that the moving body is at equilibrium when a traction force is applied to counteract the rolling friction. It can also be seen that the rolling friction is proportional to the weight of the body or the normal force acting on the tires. Table II gives C_{rr} with the rolling resistance coefficient-dimension length taken as 0.5mm for all cases.

TABLE II: ROLLING RESISTANCE COEFFICIENT [13]

Rolling Resistance Coefficient Reference Table	
C_{rr}	Tire and surface
0.001 – 0.002	Railroad steel wheels on steel rails
0.001	Bicycle tire on wooden track
0.002 – 0.005	Low resistance tubeless tires
0.002	Bicycle tire on concrete
0.004	Bicycle tire on asphalt road
0.005	Dirty tram rails
0.006 – 0.01	Truck tire on asphalt
0.008	Bicycle tire on rough paved road
0.01 – 0.015	Ordinary car tires on concrete, new asphalt, cobbles small new
0.02	Car tires on tar or asphalt
0.02	Car tires on gravel – rolled new
0.03	Car tires on cobbles – large worn
0.04 – 0.08	Car tire on solid sand, gravel loose worn, soil medium hard
0.2 – 0.4	Car tires on loose sand

The traction force is therefore given as:

$$F_T = C_{rr}R = C_{rr}W \quad (20)$$
$$F_T = C_{rr} \cdot m \cdot g$$

The traction power is the product of traction force and velocity:

$$P_T = C_{rr} \cdot m \cdot g \cdot v \quad (21)$$

Case study

The rolling resistance coefficient was taken from table II as 0.008 (Bicycle tire on rough paved road). This is the worst-case coefficient for bicycle tires. Therefore, the worst-case power associated with the traction for the Jibebe tricycle was found to be 52 W.

V. Motor and Battery Sizing

The tricycle is expected to have two power consumption modes of operation:

i. Acceleration mode: In this mode, the power drawn by the motor will be equal to the sum of all the different power requirements calculated in the previous sections.
ii. Constant velocity mode: In this mode, the power drawn by the motor will be equal to the sum of all the different power requirements calculated except for the acceleration power.

Acceleration mode is used to size the motor since it is the largest expected load: the summation of the average power (or the maximum power), the aerodynamic drag power and the traction power. The motor also has an efficiency, η_m, which should be put into consideration when calculating the motor power draw ($P_{motor\ draw}$).

$$P_{motor\ draw} = \frac{P_{ave.} + P_D + P_T}{\eta_m} \quad (22)$$

Case study

BLDC motors typically have an efficiency of 0.8 – 0.95 [14]. Table III illustrates the use of Eq. 22 with reference to the Jibebe tricycle

TABLE III: VARIOUS POWER VALUES OBTAINED FOR JIBEBE TRICYCLE

P_{ave}	P_D	P_T	η_m
165	30	52	0.8

The motor power draw is found to be equal to 309 W. This power informs on the worst-case power drawn by the motor from the supply when the tricycle is in its acceleration mode. The motor power rating should be greater than the motor power draw to avoid damage of its windings.

Once the motor is chosen by the designer, the battery can be sized with knowledge of the motor voltage (V_m) rating.

$$I_m = \frac{P_{motor\ draw}}{V_m} \quad (23)$$

The battery capacity in Ah is calculated by taking the product of the motor current and the required operating time (t_{op}).

$$\text{Battery capacity} = I_m \cdot t_{op} \quad (24)$$

Case study

For Jibebe, a 48 V, 500 W motor was chosen. It was desired that the tricycle operates for four hours. The motor current was calculated as 6.4 A from the worst-case load of 309 W and the battery capacity was found to be 26 Ah.

VI. Conclusion

To apply this method to size the battery and motor of a tricycle, the four main problems presented in section I of this manuscript have been systematically tackled. Each of the problems is tackled in sections II–V and a summary of the most important information is tabulated in Table IV:

TABLE IV: SUMMARY OF THE PROCEDURE AND EQUATIONS

	Problem/Objective	Necessary Equations		
1	Acceleration power	P_{inst}	P_{max}	P_{ave}
	Constant acceleration	Eq. 8	Eq, 9	Eq. 10
	Exponential acceleration	Eq. 14	Eq. 16	Eq. 17
2	Aerodynamic drag power	Eq. 19		

3	Traction power	Eq. 21
4	Battery and motor sizing	Eq. 22, Eq. 23, Eq. 24

Component sizing is necessary to maximize efficiency, reduce costs, enhance vehicle safety and maximize space utilization. It also gives a specific starting point for e-vehicle design which would otherwise be very difficult to validate.

Calculation of $P_{motor\ draw}$ to size the components can be done using either the average acceleration power or the maximum acceleration power. The advantage of sizing with average acceleration power is that the components are sized based on the most frequent load requirement. The components are therefore smaller, more efficient and cheaper. In practice, the maximum acceleration power is rare and occurs only for an instant. However, the merit of sizing with maximum acceleration power is that the system is safer and less prone to damage due to larger acceleration power draws. The Jibebe project used the average acceleration power to size its components.

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